

Correction du devoir n°7 - 15.

Ex 1: 1) $\frac{23\pi}{4} = \frac{24\pi - \pi}{4} = 6\pi - \frac{\pi}{4}$ la mesure principale est $(-\frac{\pi}{4})$.

2) $\frac{-20\pi}{3} = \frac{-18\pi - 2\pi}{3} = -6\pi - \frac{2\pi}{3} \rightarrow (-\frac{2\pi}{3})$

3) $\frac{37\pi}{8} = \frac{32\pi + 5\pi}{8} = 4\pi + \frac{5\pi}{8} \rightarrow (\frac{5\pi}{8})$

4) $\frac{-41\pi}{6} = \frac{-36\pi - 5\pi}{6} = -6\pi - \frac{5\pi}{6} \rightarrow (-\frac{5\pi}{6})$

Ex 2: A) $\sin(x+\pi) + \cos(x+\frac{\pi}{2}) + \sin x - \sin(-x)$
 $= -\sin x - \sin x + \sin x - (-\sin x) = 0$

B) $\cos x - \cos(x-\frac{\pi}{2}) - \sin(x-\pi) + \cos(\pi-x)$
 $= \cos x - \sin x - (-\sin x) - \cos x$
 $= -\sin x + \sin x = 0$

C) $\cos \frac{\pi}{7} + \cos \frac{9\pi}{14} + \cos \frac{8\pi}{7} + \cos \frac{23\pi}{14}$
 $= \cos \frac{\pi}{7} + \cos \frac{9\pi}{14} + \cos(\frac{\pi}{7} + \pi) + \cos(\frac{9\pi}{14} + \pi)$
 $= \cos \frac{\pi}{7} + \cos \frac{9\pi}{14} - \cos \frac{\pi}{7} - \cos \frac{9\pi}{14} = 0$

D) $\sin \frac{\pi}{5} - \sin \frac{4\pi}{5} + \sin \frac{6\pi}{5} + \sin \frac{11\pi}{5}$
 $= \sin \frac{\pi}{5} - \sin(\pi - \frac{\pi}{5}) + \sin \frac{6\pi}{5} + \sin(\frac{6\pi}{5} + \pi)$
 $= \sin \frac{\pi}{5} - \sin \frac{\pi}{5} + \sin \frac{6\pi}{5} - \sin \frac{6\pi}{5} = 0$

Ex 3: 1) $\cos x = \cos\left(\frac{-2\pi}{3}\right)$ sur $[0; 3\pi[$

$\Leftrightarrow \begin{cases} x = \frac{-2\pi}{3} + 2k\pi \\ \text{ou} \\ x = \frac{2\pi}{3} + 2k\pi \end{cases} \quad k, k' \in \mathbb{Z}$

$S = \left\{ \frac{2\pi}{3}; \frac{4\pi}{3}; \frac{8\pi}{3} \right\}$ (1)

2) $\sin x = -\frac{\sqrt{3}}{2}$ sur $]-\pi; \pi]$

$S = \left\{ -\frac{2\pi}{3}; -\frac{\pi}{3} \right\}$ (0,5)

3) $\cos\left(x + \frac{\pi}{4}\right) = \cos\frac{\pi}{5}$ sur $[0; 4\pi[$

$\begin{cases} x + \frac{\pi}{4} = \frac{\pi}{5} + 2k\pi \\ \text{ou} \\ x + \frac{\pi}{4} = -\frac{\pi}{5} + 2k'\pi \end{cases} \quad k, k' \in \mathbb{Z} \Leftrightarrow \begin{cases} x = -\frac{\pi}{20} + 2k\pi \\ \text{ou} \\ x = -\frac{9\pi}{20} + 2k'\pi \end{cases}$

$S = \left\{ \frac{31\pi}{20}; \frac{39\pi}{20}; \frac{71\pi}{20}; \frac{79\pi}{20} \right\}$ (1,5)

4) $\sin 2x = \sin\frac{\pi}{4}$ sur $]-\pi; \pi]$

$\Leftrightarrow \begin{cases} 2x = \frac{\pi}{4} + 2k\pi \\ \text{ou} \\ 2x = \frac{3\pi}{4} + 2k'\pi \end{cases} \quad k, k' \in \mathbb{Z} \Leftrightarrow \begin{cases} x = \frac{\pi}{8} + k\pi \\ \text{ou} \\ x = \frac{3\pi}{8} + k'\pi \end{cases}$

$S = \left\{ -\frac{7\pi}{8}; -\frac{5\pi}{8}; \frac{\pi}{8}; \frac{3\pi}{8} \right\}$ (1,5)

5) $\cos x > \frac{-\sqrt{2}}{2}$ sur $]-\pi; \pi]$

$S =]-\frac{3\pi}{4}; \frac{3\pi}{4}[$ (1)

6) $\sin x \geq -\frac{\sqrt{3}}{2}$ sur $]-\pi; 2\pi]$

$S =]-\pi; -\frac{2\pi}{3}] \cup \left[-\frac{\pi}{3}; \frac{4\pi}{3}\right] \cup \left[\frac{5\pi}{3}; 2\pi\right]$ (1)

7) $\sin^2 x = \frac{1}{2}$ sur $[0; 2\pi[$

$\Leftrightarrow \sin x = \frac{\sqrt{2}}{2}$ ou $\sin x = -\frac{\sqrt{2}}{2}$

$S = \left\{ \frac{\pi}{4}; \frac{3\pi}{4}; \frac{5\pi}{4}; \frac{7\pi}{4} \right\}$

(1)

8) $2\cos^2 x + \cos x - 1 = 0$ sur $]-\pi; \pi]$

$\Leftrightarrow 2x^2 + x - 1 = 0 \quad x \in [-1; 1]$

$\Leftrightarrow (x+1)(2x-1) = 0$

$\Leftrightarrow x = -1$ ou $x = \frac{1}{2}$

$\Leftrightarrow \cos x = -1$ ou $\cos x = \frac{1}{2}$

$S = \left\{ -\frac{\pi}{3}; \frac{\pi}{3}; \pi \right\}$ (2)